

Quantitative and Qualitative Aspects of the
Spread of Methicillin Resistant
Staphylococcus aureus (MRSA)
The Austin-Kakehashi-Anderson Model Revisited

Invitation by Bernhelm Booss-Bavnbek

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Thanks to

Jing Guo, Qian He, Jing Lu, Yang Wang, and Manyu Yue
of the Roskilde International Science Bachelor Program

Outline

- 1 Motivation
 - Spatial and Temporal Differences
 - New microbiological evidence
 - Math Potential
- 2 The A-K-A Compartment Model
 - Structure and Assumptions
 - Basic Strength and Weakness

Geographic Differences

Noteworthy puzzles:

- Differences by country
 - **High prevalence** countries (UK, Japan ...)
 - **Low prevalence** countries (DK, NL ...)
- Regional and local differences
 - **Geographic distribution** of clone ST80-IV in DK
 - Differences between hospitals

Changes In Time

Noteworthy puzzles:

- National level
 - Steady **levelling off** in Britain (on high level)
 - **Exponential rise** in Denmark (on low level)
- Level of institution (e.g., single hospital)
 - Internal spread of clones
 - **Imported infections**

New microbiological evidence

- [Fa05] Detailed epidemiological and molecular analysis of the 81 MRSA infections identified in Denmark in 2001
- Smaller number and greater genetic portrait variability of HA-MRSA compared with CO-MRSA
 - Of the HA-MRSA variety of 28 clones of different profile, only one popped up 5 times and four twice. All other clones were single events
- [Hu06] Amoeba promote persistence of epidemic trains of MRSA
- [Li05] Biological cost of single and multiple resistance mutations
- [Mu06] First direct documentation of (rapid) conjugal transfer between gram-negative donor and gram-positive recipient bacteria in situ

Aims of Mathematical Modeling

What mathematical models can be good for:

- **Description**
 - Point to parameters (influences)
 - Estimate parameters
 - Describe typical developments
- **Prediction**
 - Saturation level
 - Slope of development
- **Prescription**
 - Focus on infection control measures
 - Antibiotic consumption
 - Best hospital practice
 - Other measures to be taken
 - Comparative evaluation of control measures
 - Dimensioning and timing of control measures

Character of Models

Mathematical models are different:

- **Ad-hoc models**
 - Predictive power when tuned properly
 - No theoretical basis
- **Theoretically based models**
 - Strong explanatory power
 - In science: exceptional
- **Metaphors**
 - Imaginative power
 - Totally misleading when taken literally
 - Only applicable for excluding erroneous perceptions

Susceptible and Sensitive 1

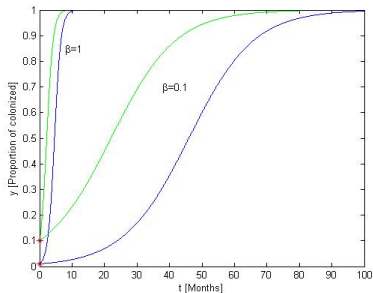
No recovery, no treatment, neither in- nor outflow

$x(t)$ Susceptible

$\beta xy \downarrow$

$y(t)$ Sensitive

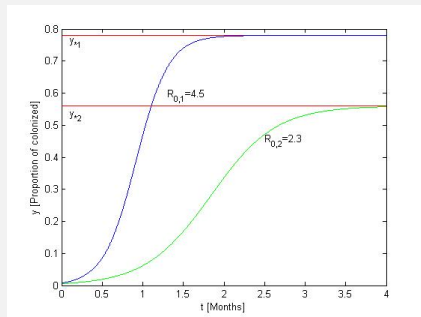
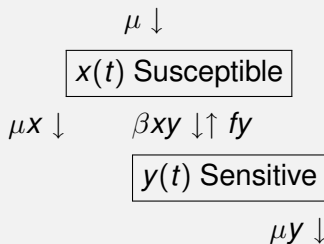
β Transmission rate



- 1 $\Delta y \propto y$, i.e., **unlimited capacity**
- 2 No separate compartment for **amoeba hosts** and **implants**
- 3 No distinction between active and less **active propagators**
- 4 No distinction between **clones** nor **colonization** vs. **outbreak**

Susceptible and Sensitive 2

No treatment, but recovery and in- and outflow



f Recovering rate, $R_0 = \frac{\beta}{f + \mu}$ Reproductive ratio,

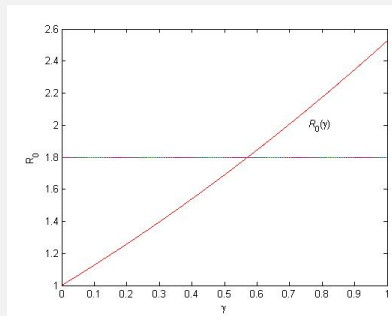
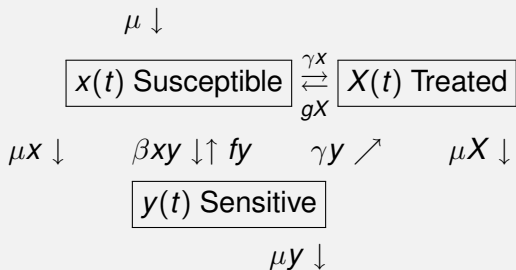
$y_* = 1 - 1/R_0$ Prevalence (if $R_0 > 1$)

5 Inflow not colonized

6 No groups of different duration of carriage $1/f$

Susceptible, Sensitive, and Treated

With antibiotics, but without resistant bacteria

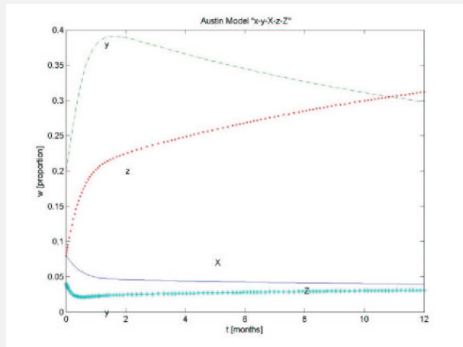
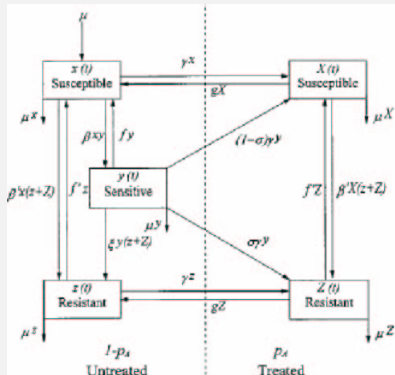


γ Prescribing rate, $1/g$ Treatment duration,

$R_0(\gamma) = (1 + \frac{\gamma}{\mu+f})(1 + \frac{\gamma}{\mu+g})$ Critical reproductive ratio

Susceptible, Sensitive, Treated, and Resistant 1

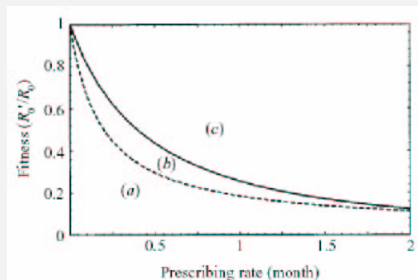
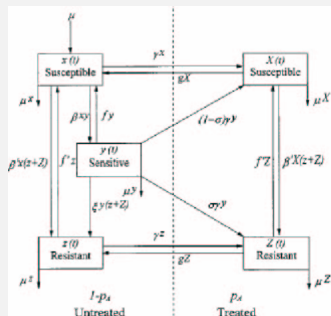
With antibiotics and resistant bacteria, see [AuKaAn97]



σ Plasmid transfer rate from natural environment under antibiotic selection pressure,
 β', ξ Transmission rates, f' Recovering rate

Susceptible, Sensitive, Treated, and Resistant 2

Endemic regions: (a) all-sensitive (b) coexisting (c) all-resistant



- 7 **Fitness cost** coded in $\beta - \beta'$, $\beta' - \xi$, $f - f'$, summarized partly in $R'_0(\gamma)/R_0(\gamma)$
- 8 No spontaneous **back transfer** from z to y
- 9 **Plasmid transfer** $\propto \gamma y$

The Austin-Kakehashi-Anderson Model, Revisited 1

Strength

- 1 Asymptotic results ($t \rightarrow \infty$)
 - Explains **S-shaped** (logistic) spread in observations
 - Indicates chances of **eradicating** MRSA due to fitness cost
 - Warns against **all-resistant endemics**
- 2 Explains **differences** by country and hospital due to best practice in hospital (β) and antibiotic prescription (γ) and opens for quantitative evaluation of **health administration measures**
- 3 Elaborates the decisive role of **plasmid transfer**
- 4 **Predictive power** depends on quality of data for estimating parameters and initial conditions

The Austin-Kakehashi-Anderson Model, Revisited 2

Weakness

- 1 $\Delta y \propto y$, i.e., **unlimited capacity** !!!!!!!!!!!!!!!!!!!!!!!!!!!!!
- 2 No separate compartment for **amoeba hosts** and **implants**
- 3 No distinction between active and less **active propagators**
- 4 No distinction between **different clones**
- 5 No distinction between **colonization** and **outbreak**
- 6 **Inflow not colonized** !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
- 7 No groups of different **duration of carriage** $1/f$
- 8 **Fitness cost** coded in $\beta - \beta'$, $\beta' - \xi$, $f - f'$, summarized partly in $\mathcal{R}'_0(\gamma)/\mathcal{R}_0(\gamma)$
- 9 No spontaneous **back transfer** from z to y
- 10 **Plasmid transfer** $\propto \gamma y$

Summary

The AKA compartment model

- seems to be a useful **ad-hoc model** and versatile for analyzing MRSA spread in **high prevalence** situations;
- is less appropriate for analyzing **geographic spread**;
- is not applicable at all in **low prevalence** situations.

Outlook

- What **Danish data** do exist; are available; can be used for re-modeling the MRSA spread in DK?
- What **discrete** or **stochastic** models should replace the AKA compartment model in low-prevalence situations?

For Further Reading I

-  D. A. A. Ala'Aldeen og K. Hiramatsu (eds.).
Staphylococcus Aureus - Molecular and Clinical Aspects.
Horwood Publishing, Chichester, 2004, xiv + 267 pp.
-  Ch. Walsh.
Antibiotics - Actions, Origins, Resistance.
American Society for Microbiology - ASM Press,
Washington D.C., 2003, x + 335 pp.
-  D. J. Austin, M. Kakehashi, R.M. Anderson.
Proc. R. Soc. Lond., B 264/1388 (November 1997):
1629–1638.
-  N. A. Faria et al.
J. Clin. Microbiol., 43/4 (April 2005): 1836–1842.

For Further Reading II



S.A. Huws et al.

Envir. Microbiol., 8/6 (June 2006): 1130–1133.



P.K. Lindgren et al.

Antimicrob. Agents Chemother., 49/6 (June 2005):
2343–2351.



S. Musovic et al.

Appl. Envir. Microbiol., 72/10 (October 2006): 6687–6692.

For Further Collaboration (First Ideas) I

Contacts, gathering, Danish or international workshop?

Molecular biologists and medical doctors:

- International
 - ...
 - ...
- Denmark
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For Further Collaboration (First Ideas) II

Contacts, gathering, Danish or international workshop?

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